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ABSTRACT

This paper examines the cognitive structures and processes that mediate mathematical and scientific ability. Ability is divided into achieved abilities and precursor abilities. Identified concepts in the area of achieved ability include expertise, understanding, and problem-solving. Other abilities can be seen as precursors to such achieved abilities, and can offer some prediction and explanation of why those achievements came about. Candidate precursor abilities include general intelligence, multiple intelligences, the triarchic theory of intelligence, and developmental level. Implications for educational practice and research are discussed, focusing on selection of talent and instructional practice. Ruth Day offers a reaction to the paper, titled "Alternative Representations: A Key to Academic Talent?." The reaction paper describes research which found that speed and accuracy in learning can be affected by the method used to represent concepts in learning, such as two-dimensional forms of three-dimensional molecules, computer cells, and computerized text editing commands. Edward Zigler offers another view in "Cognitive Theory of Academic Talent," which discusses the importance of the social and biological context of cognitive development, rate of cognitive development, creativity versus intelligence, motivation, competitiveness, etc. (JDD)

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THE COGNITIVE ROOTS OF SCIENTIFIC AND MATHEMATICAL ABILITY

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A few years ago, Ted and his father sat at friendly odds across the dining room table, eyeing a last piece of pie. As an adult, his father figured it was appropriate to be as fair as possible. No pulling rank here. So his father said, "I'll cut. You choose whichever half you want. You can choose the bigger half or the smaller half, whatever you feel like."

Ted tilted his head and narrowed his eyes. He was out to score. "If one piece is bigger than the other," he said triumphantly, "it's not *half!*"

Ted had a point. He also had much more insight into what a half was than one might expect. Those familiar with the trials and travails of mathematical education will know that youngsters commonly take a rather lax view of the partitioning of things into fractional parts. They often don't realize the importance of halves or thirds or fourths being of equal size, in order to be true halves, thirds, or fourths in the mathematical sense. It was nice to see that Ted was on top of this matter.

But what is it to be on top of such matters? To put the matter informally, what do you have to have in your head to understand about equal parts in fractions, and about the many other scientific and technical concepts where complexities and subtle distinctions need to be recognized? To put the matter formally, what sorts of cognitive structures and processes mediate mathematical and scientific ability?

There are at least two sides to that question. In part, ability is an achievement. Ted could make the distinction he did because of things he had already learned. In part, then, to ask about the roots of scientific and mathematical ability is to inquire about what cognitive structures and processes people acquire that constitute their understanding and capacity to solve problems in a discipline. Those structures and processes make up what might be called *achieved ability*.

However, to ask about the roots of scientific and mathematical ability is also to inquire about the capacities of people that allow them to reach a certain level of achievement. What sorts of structures and processes are the precursors of a certain level of achievement, or perhaps a certain pace of achievement, which might be fast, or slow, or average in comparison with that of other individuals? To ask this question is to ask about *precursor abilities*, those abilities that lead on to others.

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So what achieved abilities and precursor abilities underlie mathematical and scientific performance? Certainly no comprehensive and definitive answer can be offered here. Indeed, to ask such a question is to engage almost every aspect of a thorough going understanding of mind. But the impossibility of the answer one ideally would like is no barrier to an effort to cast a broad net into the sea of contemporary ideas about cognition, cognitive development, and intelligence, to discover what fish may be caught and how well the catch informs the questions asked.

Part I: Achieved Abilities

The overt signs of scientific and mathematical achievement are obvious enough. At the academic level, the individual knows the subject matters in question and can handle textbook problems with efficiency and some insight. The individual, even as a student, may also exhibit signs of creative work: a new theorem, a new perspective on a physical process, an innovative computational algorithm. In professional life, we look to this individual for more of the same. The individual who has in some broad sense mastered a scientific or mathematical domain should evince signs of that mastery in professional work, perhaps not radical breakthroughs, although such are welcome, of course, but at least productive and insightful application. Recalling Thomas Kuhn's distinction between normal science and scientific revolutions, one would expect, at least, the signs of good scientific practice within a paradigm (Kuhn, 1962).

However, all this is but a surface characterization of achieved ability. To get at the cognitive roots of achieved ability, one has to ask a deeper question: What cognitive structures and processes undergird it? For instance, if an individual proves to be an agile problem solver, what cognitive structures and processes equip that person to be so? We pursue this theme under three headings--expertise, understanding, and problem solving--before turning to the second major issue to be addressed: What precursor cognitive structures and processes pave the way toward mastery of a scientific or mathematical domain?

Expertise

Expert Behavior

Ted's father tells Ted to choose whichever half he wants. Instantly Ted responds, "If one piece is bigger than the other, it's not *half!*" On a small scale and in a limited way, Ted's rapid and apt reaction is a symptom of what contemporary psychology has come to call *expertise*.

In recent years, psychologists have become interested in good performance within domains, such as mathematics, physics, chemistry, or computer programming. They have studied experts, sought to characterize expert behavior, and constructed models of the psychological mechanisms that mediate that behavior. What, then, characterizes expert behavior? Among the most conspicuous attributes are the following:

Quick, recognition-like orientation to the "deep structure" of problems. Studies of chess players have demonstrated that chess masters orient very quickly

to the potentials of a chess position, often generating a likely move within a few seconds. Of course, chess masters involved in serious play think through a number of other alternatives with care. But the move first conceived frequently survives the further search as the best candidate (de Groot, 1965; Chase & Simon, 1973).

Similarly, research in the domain of physics shows that experts, when asked to sort physics problems into categories, are able to sort problems quickly according to large functional units that reflect appreciation of the physics concepts that impact on the problem, e.g., forces and energy (Chi, Feltovich, & Glaser, 1981; Larkin, McDermott, Simon, & Simon, 1980b). In contrast, novice physics students tend to break problems down into much smaller, less functional chunks, and are inclined to categorize problems less effectively in terms of the objects specified in the problem, for example pulleys, wheels, or inclined planes.

Likewise, in the field of computer programming, it has been found that expert programmers employ chunks that represent functional units (Adelson, 1981; Ehrlich& Soloway, 1984; Rich, 1981; Shneiderman, 1976). Soloway and colleagues categorize these chunks as programming plans (schemata) and rules of programming discourse that enable the programmer to comprehend programs quickly and efficiently (Ehrlich & Soloway, 1984; Soloway & Ehrlich, 1984).

Excellent memory for typical situations, but not atypical ones. A second trait, characteristic of expert performance, is an extraordinary memory for typical situations, accompanied by normal recall for atypical situations. In the chess studies cited above, master chess players were able on their first attempt to reconstruct with greater than ninty percent accuracy naturally occurring layouts of pieces on a chess board that they had seen for only five seconds. In contrast, weaker players exhibited only a forty percent success rate on their first attempt (deGroot, 1965). However, when the chess pieces were randomly assigned to locations on the board, the master players performed poorly, at roughly the same level as novices.

Building on the evidence of experts' memory for typical situations, Soloway and Ehrlich devised a study in which expert computer programmers were presented with two types of programs: those that were constructed of typical programming plans and others that were executable but did not conform to typical patterns of code (Soloway & Ehrlich, 1984). The programs were presented three times, each time for 20 seconds. On the first trial, subjects were asked to recall as much of the program as possible. On the second and third trials, they were asked either to add to their original recall or to change any part of their recall that they felt was in error.

The results paralleled those from studies of chess. After the first trial, 42 percent of the critical lines in the plan-like programs were recalled, while only 24 percent of the critical lines in the unplan-like programs were recalled. In addition, when programmers were working on unplan-like programs, they first typically incorrectly recalled plan-like answers, and only later changed their answers to match what was actually being shown in the program.

Forward reasoning to solve problems. A third attribute of expert performance is "forward reasoning." In a series of studies with expert and novice physics students, Larkin and colleagues have demonstrated that throughout the solution process experts tend to work "forward," from givens toward unknowns (Larkin, 1982, 1985; Larkin, McDermott, Simon, & Simon, 1980a,b). Principles are invoked when they can be used to find a new quantity. Thus, experts generally start with equations that involve mostly known quantities. Novices, on the other hand, typically work "backward" from the target unknown to the givens, employing a general means-ends strategy applied to stock equations.

The Mechanism of Expertise

Before the research on expertise, it was widely assumed that good performance in a domain reflected general cognitive abilities of some sort. To be sure, the chess master or skilled physicist had considerable knowledge of the domain. But there was not all *that* much knowledge to have: a few concepts, a few rules, a few equations. What made the difference between truly skilled and mediocre performance was thought to be general cognitive abilities of some sort. Perhaps, for example, the chess master benefited from a superb visual memory that allowed "thinking ahead" effectively. Perhaps the physicist benefited from an ability to think via abstract symbol systems such as algebra.

But this picture of expertise does not suit at all the profile of expert behavior sketched above. Why should experts respond so reflexively when one might expect them to reason things out more? Why should experts remember typical domain situations well but not atypical ones, if their performance depended on general cognitive abilities? Why would they not reason backward from solution needs to available information, which seems to be a highly general and straight forward solution strategy?

These puzzles led investigators to posit quite a different picture of the mechanism of expertise. It's been argued that expertise depends upon an extensive knowledge base of domain-specific schemata accessed by a recognition-like process(e.g. Chase & Simon, 1973; Glaser, 1984; Newell & Simon, 1972; Rabinowitz &Glaser, 1985). Cognitive strategies are learned within a particular context such as chess or algebra or computer programming. Thus, for example, in physics a general means-ends strategy is overridden in favor of a more context-specific scientific representation of the problem, one that allows the expert to access major principles relevant to the solution (Larkin, 1985).

Furthermore, research on expertise reveals that an extensive period of time and practice is needed to develop the repertoire indicative of expert performance in a field. Some researchers estimate that to qualify as an expert in a given domain one must accumulate about 50,000 schemata, and that such knowledge generally takes at least 10 years to acquire (Simon, 1980; Hayes, 1981, 1985). Hayes' (1985) investigation of seventy-six famous composers suggests that even gifted individuals in a field require about 10 years of concentrated involvement before they can create significant contributions to their field.

Beyond Expertise: Flexible Thinking

Some researchers have suggested that the notion of a large "compiled" knowledge base offers a sufficient account of good performance in a domain. But there are reasons to be suspicious of this claim *a priori*. The characteristic profile of expert behavior has been built on studies where experts and novices confront problems that are thoroughly typical of the domain in question, "textbook" problems one might say. But what happens when experts confront problems that are not so typical of their domains? Not problems that fall outside the domain, nor even problems that are necessarily very technical, but simply ones more off to the side of conventional practice. The question is certainly relevant, because, after all, it is the job of experts in a domain to go beyond the narrow circle of textbook problems and cope with novel problems.

Some information on this question comes from the work of John Clement, who has investigated how people well-acquainted with physics respond to a typical problems (Clement, 1982; Clement, in press). For instance, one question Clement has employed asks whether a spring of large diameter is more or less springy than one of small diameter, made of the same sized wire, and exactly why. Most people quickly conclude that the spring of large diameter is less springy, but the 'why' is more subtle. Unless you happen to have studied the mechanics of springs beyond the matter of spring constants, you probably do not know. You have to figure it out. The key insight is that the restorative force of springs derives from torsion of the wire, rather than any bending of the wire. Many people with professional level expertise in physics do not achieve this insight.

When experts tackle such untextbookish problems, what happens? According to Clement's research, "Everything" is the not too exaggerated answer. They look to kinesthetic intuitions. They try limiting cases arguments. They make far reaching analogies to other contexts. They try equations to clarify a point. In other words, the neat profile of an expert solving a domain-typical problem disappears, displaced by a wide ranging process where recourse to typical academic domain-specific knowledge is mixed in with all sorts of other recourse to general knowledge and general problem solving strategies.

To the extent that an expert can conduct this encounter with atypical problems well, once might say that the expert manifests "flexible thinking." It's not that the expert's rich domain-specific knowledge base is left behind. It figures constantly in the ongoing process. Rather, the point is that all sorts of other resources are brought to bear. A symbiotic relationship appears between typical expert knowledge and other more removed or more overarching kinds of knowledge.

Flexible thinking, in our view, has a lot to do with *understanding* the domain in question and related domains, as well as understanding one's own capacities. In a broad sense of expertise, one could say that expertise includes any understanding needed. But in a narrow sense of expertise, emphasizing only the reflexive responsiveness to typical problems, it is plain that expertise does not entail understanding. Indeed, a student easily could be an expert textbook problem solver and fall prey to all sorts of misconceptions that betray fundamental

misunderstandings. With the point in mind that understanding is not quite the same as expertise, in the narrow sense, we turn to discussing the nature of understanding.

Understanding

Understanding of Mathematics and Science as a Notable Achievement

Understanding of any subject matter is, of course, an achievement to be cherished. However, research over the past two decades on students' understanding of scientific and mathematical concepts has underscored how special and rarely won that achievement is in these technical domains. We refer here to the extensive research on students' science and mathematics misconceptions, which has demonstrated repeatedly that large numbers of learners remain subject to serious misunderstandings of key concepts, even after considerable direct instruction and even after developing significant competency with conventional numerical and algebraic textbook problems. For example, Ted's "It's not *half*!" suggested that Ted had straight what all too many youngsters have bent out of shape: It's common for children asked to divide a picture of a circle or rectangle into fractional parts to divide unevenly, as though the comparative size of the parts mattered naught.

Our agenda in this paper does not require a close review of the literature on misconceptions, but a few highlights are worth mentioning. Much literature in the domain of physics demonstrates that even students who have developed a high degree of technical problem-solving skill in dealing with textbook problems exhibit gross misconceptions when presented with tasks that do not suit the typical equation-cranking approach (Chi, Feltovich, & Glaser, 1981; Chi, Glaser, & Rees, 1982; Clement, 1982; Larkin, McDermott, Simon, & Simon, 1980b; McCloskey, 1983).

As an example, consider the "motion implies force" misconception (Clement, 1983). Clement has collected data showing that many students have a significantly different view of certain features of physics than those of Newtonian physics. When presented with a situation in which a constant force is applied to an object, a trained physicist will think of this as producing a constant acceleration in the same direction as the force. Such thinking acts as a "conceptual primitive" that allows for learning many higher-order principles in physics. However, the novice, working from an intuitive model, structures the situation differently. Proceeding from a phenomenological perspective, the student takes into account that in the real world, where friction is present, it is necessary to push on an object in order to keep it moving. Friction is often not considered by the novice to be a force. As a consequence the belief may persist that continuing motion, even at a constant speed, implies the presence of a continuing force in the same direction, as a necessary cause of motion. Clement notes that the misconception shows up in a variety of problem situations, and commonly persists even after students have completed a course in mechanics.

Such findings as these underscore the point that simply understanding the basic concepts in a scientific or mathematical domain is no mean achievement. Then, what sorts of mental structures constitute such an understanding? A

familiar, and important answer from many directions in contemporary psychology says, "mental models."

Mental Models as Structures of Understanding

"Mental models" are commonly proposed as important mediators of understanding (cf. Gentner & Stevens, 1983; Johnson-Laird, 1983). To understand, for instance, equal-sized parts in fractions, Newton's laws, or how a computer works, requires having some sort of mental representation that in a qualitative and often dynamic way symbolizes the circumstances. A case in point is the work on Newtonian microworlds by White and Horwitz (1987). They have developed a computer environment that leads students through a number of experiences with simulated Newtonian motion, aiming to remake students' naive intuitions as well as prompt them to think analytically about the Newtonian world. Use of this microworld has been shown to reduce students' misconceptions.

Another case in point is research by the authors and their colleagues on a "metacourse" for enhancing high school students' understanding of BASIC (Perkins, Schwartz, & Simmons, in press; Schwartz, Niguidula, & Perkins, 1988). The metacourse does not displace the existing curriculum, but aims to provide mental models and problem solving strategies typical programming instruction does not address. A central feature of the metacourse is a mental model called "the data factory," which depicts the computer as a factory with a few "stations," such as the keyboard, the variables area, and the program area, all serviced by an internal homunculus called NAB. The data factory model is presented through posters and an animated computer display.

The data factory model is motivated by the observations that students commonly do not seem to understand what goes on inside the computer and cannot accurately hand-execute programs, and that they often think the computer "understands" the intended functions of variables and programs. The data factory is designed to help with these difficulties by giving students a precise way to envision what happens inside a computer. Even the name NAB was selected to help: NAB is an acronym for "Not Awfully Bright," a point used to emphasize that NAB does not know the purpose of a program or variables, only a few very simple things such as where to find the current value of a variable. The programming metacourse has had considerable success in boosting student achievement in realistic educational settings.

One widely recognized problem with a mental models perspective on understanding is that the notion of a mental model is rather vague, with different authors advancing somewhat different conceptions. Some treat mental models as visual-spatial representations of dynamic systems that a person can "run" mentally (e.g., Young, 1983; de Kleer & Brown, 1983). Another approach is to speak of "frames," structures with place holders for information to be filled in from the particulars of the case at hand, with default values that supply prototypical results when information is missing (Minsky, 1975). Holland, Holyoak, Nisbett, and Thagard (1986) conceive mental models as made of "Q-morphisms," rule structures induced from experience that, even more so than frames, possess a complex default structure.

Certainly some clarification of these alternative construals of mental models would be valuable. However, despite the ambiguity, there are characteristics common to most conceptions of mental models that help in clarifying the nature of understanding. The following features seem particularly important.

Default hierarchy. Mental models typically allow for some kind of default hierarchy, by which default values fill in missing information; sometimes default values even override hard-to-perceive available information. According to Holland, et al. (1986), this helps to explain why misconceptions often persist in the face of extended instruction: Academic accounts are learned as information highly localized to the academic context, and qualitative problems of the sort that elicit misconceptions commonly trigger not the new localized knowledge but old default theories. Thus, for example, students instructed in Newtonian mechanics and given a qualitative problem are likely to default to a naive "impetus" theory of mechanics which predates by many years the academic theory.

Qualitative properties. Mental models often differ from formal treatments such as occur in algebra and physics books in highlighting qualitative characteristics of a phenomenon. While students may learn to wield formulas appropriately, they tend not to see the qualitative implications of the formulas. When faced with a qualitative problem, they often respond in terms of their naive qualitative models. Learners need more sophisticated qualitative models, but education tends not to supply them.

Simulation. Mental models generally allow for some sort of simulation or "running," in order to extrapolate implications of the model. Thus, faced with a qualitative physics problem, a student may imagine what would happen, making a "mental movie" so to speak. The simulation feature is powerful, of course, but only serves when the model that allows it matches the reality. Again, learners need sound mental models.

In sum, the notion of mental models, ambiguous though it is, contains these and other important ideas that draw a contrast with conventional textbook knowledge and help to explain shortfalls of understanding.

Beyond Mental Models: Multilayered Networks of Models

In our view, however, another problem besides the ambiguous nature of mental models plagues this perspective on the nature of understanding. Mental models as usually discussed are models of particular concepts or phenomena --Newton's first law, the behavior of an ideal gas, how a computer works inside, the way recursion occurs, and so on. Yet understanding a topic thoroughly often involves not just mental models for the key constituent concepts but a broader grasp at many levels of the matter at hand.

In an earlier paper, we have attempted to map something of the complexities of understanding scientific and mathematical concepts by defining loosely four layers of understanding (Perkins & Simmons, 1987). Just to know the rule is a matter of the content layer. But also implicated in understanding are layers involving problem solving, the epistemology of the domain, and inquiry. For a brief illustration, we elaborate a contrast between the "content" and "epistemic" frames.

Consider, for example, the idea that the parts indicated by a fraction must be equal. One aspect of understanding this, the content so to speak, is simply to know the rule and see the cases at hand as instantiations of the rule. This might involve the help of an envisioned paradigmatic case of halves, where the dividing line that generates the halves cleaves the object neatly and symmetrically in two.

But the "why" of the rule is another matter. One can easily know the rule without any appreciation for its rationale. *Epistemic* concerns deal not with the content of the ideas and principles of the domain, but rather with rationales--the standards for justification of the claims, procedures, and knowledge systems within the domain. Thus, in the case of the equal parts rule, one's epistemic deliberations lead one to ask about the nature of the content. Where would we be with fractions, for example, if one could willy-nilly pick whatever-sized parts one wanted for halves or thirds? The whole formalism with its handy algorithms would break down. To understand the consequences of *not* holding to the equal parts rule is to understand more than the rule itself, grasping something of its functional necessity.

Any discipline, including the sciences, mathematics, art, literature, or history, has an epistemic layer concerned with canons of justification. In the scientific community, for example, a strict standard of coherence in the formulation of a theory constitutes one of the primary "rules of the game" by which scientists judge validity. Small inconsistencies and mismatches with data call for careful scrutiny and may topple theories if they cannot be accounted for otherwise. However, students, not having assimilated the culture of science, often seem unimpressed by the "minor" anomalies that some instruction seeks to impress them with, in order to encourage them to revise their naive theories.

While a weak epistemic frame may undermine content learning, a strong one may abet it. Students who have developed a sense of the demand for coherence in science are more likely to take seriously the "minor" anomalies. Or, for an example from mathematics, students often exhibit "malrules"--mistaken computations such as distributed the radical sign over a sum (Resnick, 1987). Students are often encouraged to check transformations they are uncertain about with arithmetic, but tend to resist this. Novices may judge such "rules of the game" as checking with arithmetic to be rote formalism. However, an epistemological understanding of the motive can help. For example, since the grounding of rules of algebra lies in the rules of arithmetic, checking algebra against arithmetic is exactly the right step to take, a turn to the epistemological foundations of algebra. Students who appreciate this in a common sense way are perhaps more likely to make the move.

One can go much further in showing how understanding something is inherently a multidimensional pursuit. Understanding even something as simple as the equal parts rule for fractions involves a myriad of relationships with other ideas at many levels. Whatever one's particular analysis, at the least we should say that understanding something involves not one or two or three mental models for it, but a network of mental models at various levels of abstraction and with various other affiliations, that impinge on the concept in question.

It is worth noting also that navigation through this network of models is not necessarily very expert-like. It is only expert-like when one traverses very familiar regions of the network that have been overlearned and "compiled" into efficient recognitional responses. But much of the interesting work of understanding occurs on the fringes of the very familiar regions, in a more halting and belabored way, more tentative and conjectureful, but often enormously rewarding.

Problem Solving

Problem Solving Heuristics

Problem solving is a key activity in science and mathematics. Accordingly, we do well to look at it as an indicator of achieved ability. The characteristics of expertise and understanding already discussed, of course, contribute to effective problem solving. But there is a side of problem solving not addressed by the themes of having a large repertoire of schematized concepts and a deeper understanding of the epistemic roots of a discipline: the art of problem solving, as it involves heuristics of problem solving.

The notion of heuristics of problem solving has been particularly well developed in the context of mathematics. The classic source here is the work of the mathematician Gyorgy Polya, who, in a two volume treatise and a popularization called *How to Solve It*, presented a forceful counterthrust to the notion that mathematical proofs and derivations arise by mental processes that mirror their methodical logical structure (Polya, 1954, 1957).

Polya celebrated instead the importance of heuristic practices in constructing proofs and derivations. He urged that the creative work of mathematics depends crucially on a sizable repertoire of pragmatic moves that, while not guaranteeing resolution in any algorithmic sense, commonly open the way to fully formal solutions. Among the heuristic moves Polya discussed were such tactics as examining problems through diagrams, considering special cases, breaking problems into subproblems, solving simpler problems resembling the problem at hand first, generalizing problems in hopes that the more general version would actually be more accessible, and more.

Polya thought that heuristics were the key to imparting mathematical problem solving skills, and other mathematicians and educators seized upon this idea with enthusiasm. Ample evidence accumulated that mathematicians indeed depended considerably on the use of heuristics. However, significant barriers stood in the way of students quickly learning heuristics and putting them to work. One difficulty, for example, was that students commonly would simply forget to try to apply heuristics while in the midst of problems. Another was that students often would not quite know how to go about instantiating a heuristic to a particular case. It is one thing to recognize that breaking a problem into subproblems is likely to pay off, but quite another to have sufficient understanding of a domain to be able to isolate subproblems (Schoenfeld, 1978, 1979).

Heuristics Plus Problem Management

Eventually someone managed to put together the several strands needed to bring heuristics into their own. Alan Schoenfeld, a mathematician intensely interested in education and cognitive science, managed to demonstrate impressive gains in college students' problem solving through interventions providing them with Polya-like instruction (Schoenfeld, 1982; Schoenfeld & Herrmann, 1982). Schoenfeld was also able to demonstrate transfer effects to problems somewhat different from those in the instruction. A further finding was that students' perceptions of problems shifted to a more expert-like pattern, in accordance with the "quick orientation" findings regarding expertise discussed earlier.

Schoenfeld's success depended on significant innovations in the handling of Polya-like heuristics. For one, Schoenfeld presented the heuristics carefully fleshed out with real mathematical contexts, and formulated with details that made clear how they would play out mathematically. For example, not just "look at special cases" but what sorts of special cases might profitably be examined was emphasized. For another, Schoenfeld offered not only heuristics but an over-arching problem management strategy that helped students to organize their attacks on problems. This strategy pressed students to orient to a problem thoroughly at the outset, check carefully at the end, monitor progress periodically, and shift directions if an approach was not paying off, and pursue other broad tactics designed to avoid a number of typical student shortfalls.

In recent writings, Schoenfeld has emphasized that good handling of matters mathematical calls for more than just heuristic repertoire, problem management, and, of course, knowledge of the domain (Schoenfeld, 1985). There are general attitudes and background beliefs regarding the mathematical enterprise that can have a substantial influence on students' practices. For example, many students harbor the belief that "if you can't solve it in five minutes, you can't solve it at all." Such a belief, of course, undermines a proactive posture toward mathematical problems and practically ensures little systematic learning of problem solving methods.

Thus, skillful problem solving in mathematics and the sciences depends on a repertoire of heuristics, good problem solving management practices, and also conducive understandings of and attitudes toward the fields of mathematics and science. Moreover, such heuristics, problem management practices, and attitudes can be inculcated to some extent.

Beyond Problem Solving: Problem Finding

Thinking of good problem solving as the heart of good performance in science and mathematics is natural. However, such a view leaves important parts of inquiry untouched. Professional practice plainly calls for considerable attention to "problem finding," the ferreting out and formulating of problems worth solving. One is reminded again of Thomas Kuhn's notion of normal science. Problem solving is entirely too much "business as usual." In fact, it is only a limited part of even "normal science," since when scientists pursue work within a paradigm a significant part of that work consists in finding new puzzles to address and sorting them out

within the boundaries of the paradigm. Biographical work on creative scientists and mathematicians offers plentiful evidence of their attention to problem finding and their flair for sorting out more from less worthwhile problems.

In contrast with problem solving, the cognitive constituents of effective problem finding have not been extensively studied. Evidence not from mathematics or the sciences, but from the creative performance of student artists, argues that one important factor is simply the allocation of substantial attention to searching out and formulating problems (Getzels & Csikszentmihalyi, 1976). Psychological models aside, mathematics educators have devised provocative approaches to engaging students in problem finding activities, which play hardly any role in the normal curriculum. Brown and Walter (1983) offer a systematic approach to problem finding that depends on such powerful questions as "what if not," as ways of transforming given propositions into fresh and unexplored ones. Schwartz and Yerushalmy (1987) offer an approach to instruction in plane geometry that casts students in the role of discoverer of theorems, organized around a piece of software called *The Geometric Supposer* that makes geometric constructions easy and so allows students to explore possibilities fluently.

Part II: Precursor Abilities

We have seen that achieved ability in a scientific or mathematical domain is quite complex. In a general philosophical sense, this should not surprise us. But in a more specific sense, we may be a little startled by the many sides of achieved ability. As reviewed above:

- The phenomenon of expertise testifies to the importance of a large knowledge base of domain specific schemata, reflexively accessed, underlying achieved ability.

- At the same time, the use of more general principles and analogies is implicated in *flexible* expertise.

- Mental models providing dynamic envisionments, default values, and related kinds of information figure importantly in understanding key concepts in a domain.

- At the same time, "one mental model per concept" will not do. Achieved ability depends on a multilayered network of mental models, including models addressing the epistemology of a domain and other matters more abstract than the nature of particular content concepts.

- Problem-solving ability depends on a repertoire of heuristics and problem management techniques, as well as on domain specific expertise and understanding.

- Beyond problem solving, problem *finding* ability has to be considered a crucial part of achieved ability in a domain.

All this sets the stage for examining the other side of the abilities question: precursor abilities. Okay, Ted knew that the good use of fractions called for division into equal parts; that particular achieved ability might involve some sort of mental model, for instance. But what sorts of abilities predating Ted's savvy [illegible]alves or, indeed, his learning about fractions at all, put Ted in a position to [illegible]e matter of equal parts straight while many other children do not? [illegible]an such an outcome be explained to any significant degree by precursor [illegible]or does one need to turn to other factors entirely, such as motivation or [illegible]ching or conducive environment?

Of course, such questions asked of an isolated achievement such as Ted's response afford little hope of plucking a good answer out of the many possible causes. But suppose we speak of achieved ability over a broader front--being good at mathematics or physics in a general way. Then, what abilities if any can be seen as precursors to such achieved abilities, offering some prediction and explanation of why those achievements came about. Such a precursor analysis evidently has great importance for our understanding of scientific and mathematical talent.

Before turning to particular proposals for precursor abilities, it is useful to envision what an ideal account of such abilities might be like. As noted, forecast is one aim. As an ideal criterion, one would particularly like forecasts that do not involve already demonstrated achievement in the domain in question, because detecting potential in a domain in advance of actual work in the domain is desirable.

Explanation is another function of precursor abilities. One would like such phenomena as demonstrated talent in a domain, insightful understanding, problem solving prowess, and so on, explained by reference to precursor abilities out of which these achieved abilities develop. Again, a good precursor explanation would not involve precursor abilities that already constituted achievement in the domain in question. After all, to explain further achievement in terms of achievement so far is to do little more than take what we've already discussed--achieved ability--and say that *it* explains more of the same. It's like explaining a mature oaktree by pointing to an oak sapling and saying, "It keeps growing in about the same shape!"

For both predictive and explanatory reasons, then, one would like to find precursor abilities with a distinct "remove" from the nature of the achieved abilities. This, as will be seen, is not an easy challenge to meet. We turn now to an appraisal of several candidate precursor abilities.

General Intelligence

Perhaps the most obvious candidate for a precursor ability is the well-known *g* of general intelligence. The aim of *g* is to serve as an index of a broad-band mental ability that manifests itself in virtually all performance contexts. Indeed, we do well to recall that this is the very basis of the construction of *g*. If one gives individuals a variety of intellectually challenging tasks drawing on various domains--language, mathematics, pictorial tasks, and so on--one invariably finds

considerable intercorrelation among performances on the instruments: Those who do well or poorly on one tend to do well or poorly on the others. The common trend can be extracted statistically, and yields the *g* measure, a numerical effort to capture whatever it is that enabled the person to perform at a certain level across the various tasks.

While *g* in itself is a mere measure, not a model of anything, models of the mechanism that *g* reflects can be constructed. For example, Arthur Jensen argues that *g* is an index reflecting the fundamental efficiency of the neurological system in processing information of any sort (Jensen, 1984). Jensen buttresses this argument with a series of experiments that demonstrate correlations between *g* as conventionally measured and performance on what is called a "choice reaction time" task. This task requires subjects to make a rapid choice between varying numbers of buttons in different conditions: two, four, or eight buttons. Taking the log base 2 of the choices yields a measure of information that proves a linear predictor of certain aspects of subjects' reaction times in responding, one reason one might agree that a fundamental characteristic of information processing is implicated.

Then does *g* offer the ideal precursor ability we want? Consider first the predictive side of the question. In some ways, *g* shows definite promise. Virtually all studies of high achievers in intellectually demanding domains have shown these talented individuals to have relatively high *g* (Grinder, 1985; Sternberg & Davidson, 1985). Accordingly, *g* affords significant predictive value in forecasting high achievement.

On the other hand, such predictions are starkly limited by a further finding: Having a very high *g* in no way guarantees truly exceptional achievement (Grinder, 1985; Sternberg & Davidson, 1985; Wallach, 1985). Specifically, within a profession, correlations between *g* and professional productivity tend close to zero. For example, professional physicists with exceptionally high *g*'s are no more productive or creative than those with lower *g*'s. One way this result is sometimes stated is to say that *g* functions like a gatekeeper to certain professions: You simply do not become a professional physicist unless you have a good *g*. But among professionals, *g* has no further predictive power. To put it another way, a fairly high *g* serves more as a necessary than as a sufficient condition for high achieved ability: One does not find such achievement without a high *g*, but a high *g* comes nowhere near guaranteeing special achievement.

Yet another qualification on the predictiveness of *g* is that *g* corrected for age, that is, IQ, is not at all an invariant trait (Humphreys, 1985). An individual's IQ can drift up or down considerably as the individual matures. Specifically, the correlation between *g* at a given point in time and *change* in *g* over some period thereafter, say a year, is about zero. That is, as *g* increases with age, a person is about equally likely to gain or lose ground relative to peers. In IQ terms, IQ is equally likely to drift up or down.

Thus, one simply cannot view a person with a normal or below normal IQ as somehow trapped by a ceiling level of intellectual capacity. That person's IQ may well rise. Likewise, one cannot view a person with high IQ as somehow supported

in intellectual performance by some floor capacity. That person's IQ may well drop.

If *g* is only a so-so predictor, what about *g* as an explainer of later achieved ability? Again, *g* or rather the mechanisms underlying *g* certainly offer something toward an explanation. For example, adopting Jensen's notion of effective information processing, we certainly find in that notion a precursor ability well removed from the particular achieved ability of a mathematician proving theorems in differential geometry or a physicist developing a theory of superstrings (Jensen, 1984).

At the same time, though, we might feel uneasy that *g* does not offer anything toward a *selective* explanation of special achievement. Certainly it seems that different individuals often display particular talents for different domains. One gifted person may incline to mathematics, another to painting,another to music. However, *g*, domain-neutral as it is, does nothing as a precursor to reflect such early leanings. It thus explains less than we would like to explain.

Why might it be that *g* is neither as predictive nor as explanatory as one would like? This is a surprisingly easy question to answer. First of all, numerous studies of talent have demonstrated that high motivation plays a crucial role in high achievement. One simply does not find prodigious development in youngsters who have not committed themselves to extensive work in a domain (Bloom, 1985; Feldman, 1986). This result makes sense in terms of the research on expertise cited earlier, with its emphasis on a large knowledge base of schemata and on the time needed to acquire such a repertoire. It also reminds us that achieved ability will vary strongly with time-on-task, and that, in turn, will vary as strongly with motivation as with a precursor ability such as *g*. Opportunity to learn, including access to special tutors, is of course another factor that has been shown to be important in developing talent and hence another factor introducing variance.

These considerations aside, a further element that may limit the relevance of *g* is the possibility that intelligence is much more multifold than monolithic. While *g* aims to compress variations in ability into a single measure and a single distributed property of the nervous system, say Jensen's information processing ability, numerous investigators have suggested that intellectual competence rests in several somewhat independent factors. The following two sections address expanded notions of intellectual competence in this spirit.

Multiple Intelligences

An approach better suited to accounting for special talents might be Howard Gardner's theory of multiple intelligences (Gardner, 1983). In his theory, Gardner criticizes the view of intelligence as a general capacity, arguing instead for a broader and more universal set of intelligences than those typically considered. In stark contrast to theories propounding the existence of a general intelligence, Gardner maintains that individuals have a number of domains of potential competence which they may develop assuming the appropriate stimulating factors are accessible.

Defining intelligence as the ability to solve problems or fashion products that are deemed of value in some cultural context, Gardner proposes a set of seven competencies or intelligences. Underscoring the point that no one list of intelligences covers the full range of cognitive capabilities, Gardner focuses his discussion on the following: linguistic intelligence, musical intelligence, logical-mathematical intelligence, spatial intelligence, bodily-kinesthetic intelligence, and two aspects of personal intelligence, intrapersonal and interpersonal. Gardner stresses that intelligences normally considered outside the traditional realm of the study of cognition, for instance, the personal or musical intelligences, in a broad sense, hold equal status to those typically held at a premium in our schooling and testing practices, namely logical-mathematical and linguistic abilities.

These seven competencies, Gardner argues, act relatively independently of one another, so that, for example, one's linguistic ability cannot be inferred from one's competence in music or logical-mathematical intelligence. While maintaining the importance of isolating competencies in discussing the structure of human cognition, Gardner notes that any adult recognized by society as particularly gifted in a certain area, say in musical intelligence, undoubtedly possesses a blend of well-developed intelligences. Thus, in addition to heightened musical talents, the musician may possess a highly developed interpersonal competence that allows for communication with an audience, as well as some bodily-kinesthetic skills that enable him or her to achieve subtle effects on a particular instrument.

Gardner argues that the current system of intelligence testing skews results in the direction of those who possess linguistic and logical intelligences. He maintains that even tests that *seem* to measure spatial or other abilities are constructed so that they primarily call upon linguistic and logical facility; accordingly, individuals who do well in these areas are more likely to perform well even in tests of spatial or musical abilities. Conversely, individuals with strengths in other areas, but not highly developed in logical or verbal skills, will not fare well on such tests. Gardner advocates the construction of "intelligence-fair" instruments that would more adequately assess the intelligences in question. He suggests, for example, that an assessment of spatial ability might involve navigation about an unfamiliar environment as opposed to the typical series of multiple choice geometric rotations.

How well does the theory of multiple intelligences speak to our search for precursor abilities? In contrast to *g*, the theory offers an explanation of special abilities in particular domains. And, as an explanatory device, the theory of multiple intelligences does indeed offer an explanation of talent removed from its playing out in a discipline. Indeed, Gardner views the seven intelligences to have to a considerable degree neurological bases, corresponding to different "organic computers" within the larger system of the brain. Gifted individuals may well be gifted in virtue of better "original equipment," although Gardner also emphasizes the impressive power of artful instruction, pointing to Suzuki violin instruction as an example.

Without engaging the technical details of Gardner's and rival theories, a reservation one might have about its explanatory power is simply that the theory of multiple intelligences cannot be viewed as established. It, along with virtually every other theory of intelligence propounded in this vexed area of the psychology

of mental competence, is quite controversial. Indeed, there is a history of efforts to break *g* down into well-defended subcomponents, including,for example, J. P. Guilford's *structure of intellect* theory, several components of which anticipate Gardner's multiple intelligences model. These efforts have been criticized for failing to make a technically good case for multiple components of intelligence (Grinder, 1985; Humphreys, 1985).

What about the quest for a *predictive* account of precursor abilities? In this regard, a problem with the theory of multiple intelligences as it stands today is the lack of instrumentation available to measure the intelligences. While Gardner himself recognizes this dilemma, advocates the development of such measures, and is engaged in such work, the gap remains and for the present, at least, limits the usefulness of the theory to deliberations of an explanatory rather than a predictive nature.

Moreover, Gardner's conception of appropriate means to develop measures of the intelligences emphasizes engaging students in activities characteristic of the domain in question, often quite rich activities. Such measures sacrifice something of the "remove" called for earlier, coming close to gauging achieved ability. The sacrifice may be appropriate. One can understand the sensibleness of Gardner's approach: What better probe of an intelligence than rich tasks from the domain in question? Still, this pushes matters toward the "sapling" model, that gauges the height of the mature tree by the height of the young tree.

Finally, such measures, if developed, might still prove problematic in much the same way that *g* does. To be sure, they would honor a greater range of human capacities. Nonetheless, as previously noted, there were good reasons why *g* appeared to function more as a necessary than a sufficient condition for high achieved ability. The same sorts of reasons seem likely to apply to measures of multiple intelligences. As discussed earlier, other intervening factors, such as motivation and opportunity to learn, affect achievement and thus would interfere with the predictive ability of Gardner's intelligences.

Gardner's own theory compounds the sources of variance, because of his reasonable insistence that a particularly able individual's performance is likely to involve the significant exercise of a portfolio of intelligences. If a fine musician's achievement involves a number of competencies, how is one to predict? Apparently not on the basis of musical intelligence alone, which would seem to be a necessary condition only. But what others might be called for? Different ones in different cases, perhaps.

The Triarchic Theory of Intelligence

Yet another contemporary view is Robert Sternberg's *triarchic theory of intelligence* (Sternberg, 1985). In the present context, we will focus on one aspect of this complex theory, its major tripartite division. Sternberg defines three subtheories--hence "triarchic"--termed the *contextual, experiential,* and *componential* subtheories. Sternberg avers that these represent contrasting aspects of intelligence and that taken together they encompass far more of what

intelligence ought to mean than the traditional *g*. A person may be more or less high in any one of the three aspects somewhat independently of one another.

The contextual side of intelligence concerns a person's adaptation to environment, including social milieu. Sternberg notes that some people may be exceptionally well adapted to intellectual environments--universities and research laboratories, for example--without necessarily being strikingly creative or extremely bright in an academic sense. They may be good organizers, good promoters of themselves and others, and build quite a reputation by knowing how to move well in the world they occupy. As Sternberg notes, in our more critical moments we may call them "operators."

The experiential side of intelligence concerns a person's ability to adapt to novel tasks and situational demands, both by insight into new situations and by efficient automatization of responses to deal with recurrent situations. In this aspect of intelligence one finds giftedness in the "great person" sense --the Einsteins, Beethovens, and others who remake fields in fundamental ways. Such individuals may be far from "operators," of course.

The componential side of intelligence concerns a person's ability to process information effectively via a number of components defined by Sternberg. Prominent among these are "metacomponents," several very general components that exercise a control function over the organism. Examples of metacomponents include "Decision as to just what the problem is that needs to be solved" (Sternberg, 1985, p. 99) and "selection of one or more representations or organizations for information" (Sternberg, 1985, p. 100). Conventional academic performance and *g* are most closely related to strong components, particularly strong metacomponents, according to Sternberg. Of course, many individuals prove strong academically without exhibiting either the flair for insight of the experiential side of intelligence or the flair for maneuvering of the contextual side of intelligence.

This does not mean, however, that these components play no role in the experiential and contextual sides of intelligence. Actually, the componential side of intelligence is less a third side than an undergirding relevant to all handling of information.

To understand Sternberg's perspective, recognizing that two of the subtheories in the triarchic theory have familiar ancestor is helpful. Roughly speaking, the componential theory is meant to be a sharper articulation of the component processes involved in conventional *g* and what might be called "academic intelligence." The experiential side is in large part an effort to map insight and creativity. There is a long and, at least, somewhat successful history of efforts to separate academic intelligence from creativity and a number of different approaches to doing so, to which the triarchic theory may be added (cf. Perkins, in press). Academic intelligence and creativity are, of course, the two sides of intelligence most directly germane to academic giftedness as normally conceived. Less precedented but also relevant to academic giftedness is Sternberg's introduction of the contextual side of intelligence.

How does Sternberg's triarchic theory fair as an account of precursor ability? In principle, the triarchic theory offers good "remove" from particular achieved ability in science and mathematics. It is not formulated in domain-specific terms, and the sorts of measures discussed by Sternberg do not require achievement in depth in particular disciplines. Accordingly, the triarchic theory offers the potential for the sort of distanced prediction and explanation that an ideal precursor account demands.

What can be said about its predictive function specifically? Here we encounter a problem somewhat analogous to that examined in the discussion of Gardner's theory of multiple intelligence. Although Sternberg does propose some measures, by and large, there is no substantial validation of them against individuals of genuine real-world achievement. Nor is there any reason to think that Sternberg's measures overall would be any *more* predictive of academic and creative performance than prior efforts to forecast such abilities.

In sum, the best assessment at present of what Sternberg's account might offer predictively is simply what tests of academic intelligence and creativity have offered in general. What tests of academic intelligence--*g*--put forth has already been discussed. Regarding creativity testing, the basic point to make in this context is that such tests have proved weakly predictive at best (cf. Wallach, 1976a,b, 1985). The best measures have been shown to be not cognitive instruments but personality profiles, and even better than that is track record in a domain (Mansfield & Busse, 1981). In other words, the available instrumentation in precursor abilities comes nowhere near the predictiveness of achieved ability during engagement with a discipline.

With this said for *prediction*, how does the triarchic theory fare as *explanation* for talent in science and mathematics? Unlike Gardner's theory of multiple intelligences, the triarchic theory in its basic structure does not offer any account of field-specific leanings. In this respect, therefore, we may feel disappointed if, indeed, we believe that field-specific leanings exist before actual engagement with a field.

In its componential subtheory, the triarchic theory presents a much better articulated conception of how academic intelligence works than conventional accounts, with an emphasis, in our view appropriate, on metacomponents that play a crucial organizing role in the deployment of mental resources. We note that Sternberg's particular choice of metacomponents lacks strong support and that one might propose many other sets of metacomponents of equal plausibility. Nonetheless, the triarchic theory offers one such organization and viewing intelligent functioning at that "metalevel" provides explanatory power lacking in more monolithic and single level concepts of academic intelligence.

The triarchic theory of intelligence also offers a theory of insightful thinking that highlights the three operations *selective encoding, selecting combination,* and *selective comparison*. These provide a framework for discussing episodes of insight in a systematic manner. However, as Perkins (in press) argues, the same framework functions just as well for discussing quite mundane contexts of information processing; a case is lacking that these operations are *distinctively* involved in insightful thinking in a noncircular way.

In sum, the triarchic theory seems to be particularly useful in explaining academic intelligence, less useful in accounting for insight and creativity, and distinctive talents.

Developmental Level

Another very different approach to seeking precursor abilities is to turn to concepts from the field of human development. Piagetian stages provide obvious candidates for precursor abilities. For example, certain concepts important in science and mathematics simply are not accessible until an individual has attained formal operations. Moreover, with formal operations available, the individual should have no great difficulty attaining these concepts.

As with Gardner's theory of multiple intelligences and Sternberg's triarchic theory, Piaget's notions of concrete and formal operations satisfy the requirement of "remove" for precursor abilities. Concrete and formal operations are conceived to be quite general cognitive structures not strongly affiliated with any particular disciplinary expertise. From this standpoint, concrete and formal operations make attractive precursor abilities.

The difficulty is that cognitive developmental research over the last two decades has pretty much discredited these Piagetian notions as originally conceived (Carey, 1985a, b; Case, 1985; Brainerd, 1983; Fischer, 1980; Fischer, Hand, & Russell, 1984). Extensive evidence has accumulated against the idea that there are sweeping stage transitions in which numerous concepts across many domains suddenly become quite accessible to the learner. The current view is overwhelmingly that there are individual developmental trajectories in different domains, strongly influenced by the structure of knowledge in the domains. Even Piaget late in his life acknowledged phenomena of this sort (Piaget, 1972). Thus, for example, a youngster who had for one reason or another received extensive exposure to and involvement with mathematical thinking might display patterns of mathematical reasoning of a fully formal character, while the same youngster might evince much less sophisticated thinking in other contexts.

This shift away from the idea of overarching developmental stages is, of course, tied intimately to the research on expertise reviewed in the first part of this document. With the recognition of the importance of a rich knowledge base in accounting for expert performance came a correlative recognition that early development of school competencies might also reflect particular knowledge more than anything else. Accordingly, some developmentalists now argue that cognitive development is *nothing but* the acquisition of various kinds of knowledge, much of it but not all of it domain specific (Carey, 1985a, b). The case can be made that developmental psychology would best inform education through cleaving closer to subject matter learning while preserving important developmental themes (Strauss, 1986, 1987).

Recent years have seen a resurgence of models of development that rescue various parts of Piaget's conception. The developmental theories of Robbie Case and Kurt Fischer are examples of these perspectives (Case, 1985; Fischer, 1980;

Fischer, Hand, & Russell, 1984). Both offer structural concepts of mental organization that illuminate the process of coming to understand concepts and processes within a domain. By and large, however, these neo-Piagetian models do not offer precursor abilities with "remove" from achieved abilities, exactly because they highlight the importance of domain-specific learning. Rather, they basically enrich our conception of what achieved abilities are like and how achieved abilities of a certain complexity and degree of integration can lead to other achieved abilities of greater complexity and integration.

The Cognitive Roots of Ability

With these ideas examined, it is appropriate to stand back and see where we are. In quest of the cognitive roots of scientific and mathematical ability, we have sought to characterize *achieved ability*--what sorts of cognitive structures and processes mediate competence in a domain? We also have sought to characterize *precursor ability*--what cognitive structures and processes "removed" from the domain of interest, here science or mathematics, predict and explain achieved ability in the domain? Let us consider what we have found out about each of these questions, beginning with the latter.

Precursor Abilities

Our analysis offers a number of "softly necessary conditions" with some "remove" from achieved ability and some explanatory power. Among these precursor abilities we find the classic notion of general intelligence, Gardner's theory of multiple intelligences, and Sternberg's triarchic theory of intelligence.

Ideally, however, one would hope for a much stronger account of precursor abilities. The "softly necessary" character of the aspects of intelligence identified makes them only predictive--and indeed explanatory--in a very partial way. Of course, other than in the case of *g*, our only evidence for the "softly necessary" character of these aspects of intelligence is our own reasoned judgment, because of another unfortunate circumstance, the lack of actual measures representing adequately either the theory of multiple intelligences or the triarchic theory. It must be added that significant controversy surrounds all these precursor theories, including *g* itself. Finally, while insight and inventiveness would naturally be of special interest in considering education for the talented, the experiential component of Sternberg's theory may not even provide "softly necessary" conditions. At least somewhat analogous efforts to measure inventiveness in cognitive ways have a very checkered history, while personality factors and, especially, track record have proved much better predictors.

It seems clear that, if one wants to move toward sufficient conditions, nothing comes close to beating the "sapling test"--actual achieved ability in the domain at a certain point in time. We can understand why in light of the complexity of achieved ability: So many dimensions and aspects are involved that achieved ability is hard to predict in a sufficient condition sense by anything else other than prior achieved ability in the same domain. One is even tempted to question whether a strong theory of precursor ability is possible. There may be

too much room for variation in individual histories and the factors important in those histories.

Achieved Abilities

If contemporary psychology offers a rather limited account of precursor ability, it serves up quite a rich account of achieved ability. Recall that the survey in Part I of this article identified at least the following elements:

- "Compiled" domain-specific repertoires of schemata.
- Flexible access to more general schemata.
- Mental models of many sorts serving understanding.
- Multilayered networks of mental models.
- Heuristic and problem management tactics.
- Problem finding abilities.

Moreover, the perspectives reviewed in Part II, whatever their contribution to an account of precursor abilities, expand our understanding of achieved ability further by introducing yet more elements to take into account. Although these elements overlap somewhat with those introduced in Part I, they are very much worth considering. Here is a partial list:

- Achieved competence in the several intelligences.
- Achieved competence in metacognitive components.
- Achieved competence in insight-related processes.
- Achieved domain-specific developmental integrations.

Implications for Educational Practice and Research

Selection of talent. The distillation of the perspectives reviewed here suggests that selection of talent should focus on early achieved ability, the "sapling" approach. Simply put, no strongly predictive measures of precursor ability are available. A number of measures could be used for screening as "softly necessary conditions," but they should not be seen as ensuring high achieved ability in the domain in question. Moreover, the "softly necessary" should be remembered: That any such measures, as with IQ, will be subject to upward and downward drift relative to peers is likely. Finally, it is important to remember that selection of an extreme group is subject to effects of regression to the mean (Humphreys, 1985); the group probably will never be quite as special as when it was selected!

In terms of research, the theories reviewed both in Part I and Part II suggest that the bases for examining achieved ability for predictive purposes might be broadened considerably. For example, one might test seemingly talented individuals not only for conventional problem solving but for problem finding. One might employ measures of insightful problem solving or of personality trends. One might check for a precocious understanding of the epistemic side of the discipline in question. One might gauge ability in an intelligence central to the discipline in question, plus correlative intelligences thought likely to make particularly important contributions.

Whether any of these probes would, in fact, increase the predictiveness of the "sapling" approach is, of course, an empirical question. However, it is easy to see how they might: by broadening the measurement process to include a number of very important factors that typically remain untapped by conventional measures.

Instructional practice. The multifaceted nature of achieved ability and the subjectness of precursor abilities to drift also have definite implications for instructional practice. The drift effect implies that, certainly, one should not take for granted continued "automatic" development of the talented or the "automatic" slow development of the less talented. A drifting system invites boosting in desirable directions. Too often, teachers and parents treat either slowness or giftedness at a certain point in a child's development as a durable trait sure to play out to a clear destiny. Such an attitude is likely to lead to undeserving both slow and gifted students.

As to the multifaceted nature of achieved ability, each facet recognized shows conventional educational practice to be all the more reductive. Most of the facets discussed--for instance problem solving heuristic, the epistemic frame, mental models--are simply not substantively addressed at all in conventional education. Instruction truly seeking to enhance achieved ability needs much greater scope. The aspects of achieved ability identified provide a useful guide to what *sorts* of scope might be helpful. One might look toward "wide spectrum education" (WISE for short!) that attends to education in the subject matters with much more respect for the many aspects of achieved ability.

"Wide spectrum education," through addressing the many aspects of achieved ability better, offers the hope of capturing individual talent better. Particular students might well prove more or less responsive to different facets of achieved ability within a discipline. Some learners might need or especially value concrete mental models of basic concepts; others, an epistemic understanding of the domain so that it seems better justified; others, problem solving management techniques, and so on. Each facet also helps the whole to make more sense, and so many more students who now find education exciting and effective might discover entry ways into achieved ability through a wide spectrum approach.

While a broad prescription for educational reform, these ideas also write their own research agenda. An instructional process as rich as envisioned here plainly runs the risk of utter impracticality. Design questions arise: What can be done for expanded range with reasonable effort? Impact questions arise: When such plans are followed, what are their effects? Innovations such as the notion of "metacourses" discussed earlier designed to bring wider spectrum education to

conventional educational settings, invite systematic research (Perkins, Schwartz, & Simmons, in press; Schwartz, Niguidula, & Perkins, 1988).

All in all, this two-part effort to examine the cognitive roots of mathematical and scientific ability has yielded a worthwhile harvest, if not a cornucopia. A good deal can be said about *achieved ability*, including the notion that early achieved ability, given learning opportunities, seems like the best predictor of later achieved ability--the "sapling test" appears much superior, its lack of remove notwithstanding. Some broad implications for educational practice and research also follow.

As to the second part, sifting from contemporary cognitive psychology a strong theory of *precursor abilities* has proved move difficult. Indeed, the available efforts in that direction may contribute more in rounding out our notions of achieved ability than in building the ideal precursor theory. Well, as the saying goes, half a loaf is better than none. And, if we are right in our speculation that an ideal theory of precursor abilities may simply not be possible, and since Ted is not sitting across the table, we are tempted to say that the half we have is the larger half.

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Discussant Reaction: Alternative Representations: A Key To Academic Talent?

Ruth S. Day

It is a tall order to understand the cognitive roots of scientific and mathematical ability. Nevertheless Perkins and Simmons (1988) have provided a useful overview of this complex problem. Central to their discussion is the contrast between two general classes of abilities, those which have already been achieved and those which are precursors to such achievement.

Achieved Abilities

In discussing achieved abilities, Perkins and Simmons reviewed three research traditions and implied how each reveals important aspects of cognitive talent. Thus, according to their view: 1) Research on expertise recognizes domain specific schemes, and talented individuals should have many such schemes and have flexible access to them. 2) Research on understanding emphasizes the role of mental models, and talented individuals should have many such models arranged in multilayered networks. 3) Problem solving research emphasizes the role of heuristic, management, and problem finding, and talented individuals should have more and better heuristic, manage the problem solving process more efficiently, and be better able to find problems as well as solve them.

Precursor Abilities

In discussing precursor abilities, Perkins and Simmons highlight four research traditions (general intelligence, multiple intelligences, triarchic theory, and developmental level) and evaluate each in terms of its ability to explain and predict talent. Unfortunately, they are not very sanguine about the accomplishments of these approaches. Given sufficient time, I would suggest another type of precursor ability, overall cognitive pattern (Day, 1977), which may predispose individuals to have different types of cognitive talents. I would also expand the achieved-precursor paradigm to study how these two types of abilities might interact with each other. Such discussion might suggest new approaches for theory, research, and educational practice. However, since this presentation must be brief, I will only discuss the concepts of alternative representations and problem finding as mentioned above.

General Approach

In the research reported below, we identified several academic problems and then translated them into carefully controlled laboratory experiments. Unless indicated otherwise, subjects were undergraduates at Duke University. Since Duke students have average combined SAT scores above 1300, they are quite academically able; however, not all would be classified as academically "talented." In all experiments, subjects were assigned randomly to the various treatment conditions. Implications of this work for understanding and fostering talent are presented at the end of this paper, along with a fairly unorthodox view of talent.

The Alternative Representations Approach

Two simple notions introduce the alternative representations approach (Day, 1988). First, all ideas can be represented in many different ways--such as sentences, lists, outlines, tree diagrams, matrices, graphs, and pictures. This is true no matter whether the content involves academic or everyday information, is simple or complex, brief or extensive in length. Second, the way we represent a given idea has clear cognitive consequences. That is, representations affect cognitive processes such as perception, memory, concept formation, comprehension, and problem solving. To illustrate these notions, this discussion includes examples from specific science domains as well as some which cut across domains (text editing, data analysis). The discussion concludes with implications for understanding the nature of talent and form modifying instructional practice.

Chemistry Molecules

Many students have great difficulty learning organic chemistry. For example, the Duke University *Chronicle* (March 27, 1984) titled an article describing this problem as, "Pre-Meds Face the Wrath of Orgo, the Killer Course." There are many reasons why organic chemistry is so difficult, including the sheer amount and complexity of the information involved. We have been studying another possibility, that the notation systems chemists use to represent three-dimensional molecules in two-dimensional form (in textbooks, on blackboards) may be part of the problem (Day, 1984). A quick look through any organic chemistry textbook reveals a wide variety of diagrams such as those shown in Figure 1. To what extent does each type of representation help or hinder understanding the molecules?

In our experiments, subjects were undergraduates who had taken one high school or college chemistry course; thus they were like those who typically enroll in organic chemistry. They handled a tinker-toy model of a new molecule we devised and learned to represent it in various standard notation systems. Then they saw two such displays and had to determine whether they showed the same or different molecules. Their performance depended on what notational system we used; one system was easy to use and led to superior performance while another was difficult and led to inferior performance. The properties of these notations are described elsewhere (Day, 1984; in preparation); for simplicity here, they are called the "easy" and "difficult" notations.

Clearly, we can make it easier or harder for students to understand chemistry molecules, simply by using alternative representations of the same information. A more provocative way to say this is that we can make students appear to have more or less "chemistry talent," simply by using specific representations. Curiously, the difficult notation we used is introduced early in organic chemistry courses and used heavily throughout the course, while the easy one is introduced much later; by that point, many students have fallen behind, become confused, or even dropped the course.

Hardware Design

Designing hardware for a computer involves a considerable amount of knowledge and skill, including physics, electronics, computing, design, and problem

solving. Yet a given project will consist of many quite small units, or "cells." Current research in our lab represents such cells in alternative ways, emphasizing their electrical or logical properties. Subjects are graduate students who have had at least one course in VLSI (very large-scale integration) but little if any experience beyond such coursework; thus they are VLSI novices--somewhat knowledgeable but inexperienced. Subjects inspect cells one at a time and try to determine their general function. Although they are highly accurate in this task, alternative cell representations affect the speed with which they can respond--and thereby understand the function of the cells. It is important to understand the function of many cells at once during normal design tasks; hence, alternative cell representations may well affect engineers' ability to develop hardware systems in both a quick and efficient manner.

Text Editing

Computer text editors enable users to modify text quickly and easily--to delete, add, modify, and rearrange units as small as single letters and as large as entire sections of manuscripts. Despite the many advantages of such systems, learning to use them is rarely a quick and easy process. However, recent research (Day, 1988) showed that representing simple computer commands in alternative ways makes it easier or harder to learn and use them. Subjects learned six simple editing commands used to move the cursor around the display screen; the commands were taken from an editing system unfamiliar to the subjects (Emacs, developed by Richard M. Stallman at the MIT Artificial Intelligence Laboratory). For example, the command *B* moves the cursor back one space. The commands and their definitions were displayed in standard list format or in a spatial format, as shown in Figure 2. Note that both formats used the exact same command letters and definitions but differed in their spatial arrangement of this information.

Subjects studied one of these displays and later solved a series of cursor-movement problems like the samples shown in Figure 2. They saw simplified computer screens consisting of dashes to indicate locations for potential characters and had to specify the common (s) needed to move the cursor (the small filled box) from its current location to a new location (indicated by the asterisk). For example, the current answer to the first sample problem in Figure 2 is F, which moves the cursor one location to the right. Subjects who studied the spatial representation were more accurate in this task than those who studied the list. Furthermore, they were more efficient. For example, the second sample problem in Figure 2 can be solved with either four keystrokes (*BBBP* or *PBBB*) or two keystrokes (*AP* or *PA*); subjects who studied the spatial format used fewer keystrokes on such problems. Thus, simple alternative representations of the same commands affected subjects' ability to use the information in an accurate and efficient manner. The spatial representation is more effective because it explicitly reflects the way the information is to be used--to move the cursor in various directions and across various distances. In general, it is important to match the form of a representation to both the nature of the information itself and to the nature of the task to be performed (Day, 1988).

Data Analysis and Problem Finding

Although most students study science in high school and college, can they think scientifically? Surely they learn about taxonomic systems in biology, formulas in physics, and the periodic table in chemistry. But can they take some simple evidence, analyze it, find its regularities, form and test hypotheses, and suggest useful future work? To study these questions, we give students a small set of data from a simple experiment and ask them to extract some basic results (Day and Diaz, 1988). Providing the same data in alternative representations affects their ability to solve simple problems. More interestingly, though, such representations also affect their ability to find and then solve more subtle problems embedded in the data. Subjects in the favorable representation groups, then, look as if they possess greater "scientific talent." In this and all experiments reported here, assignment to representation conditions is random, so *a priori* individual differences in scientific talent cannot account for these results. Thus, certain representations may potentially be used as a powerful tool to facilitate scientific thinking.

Overview

In the experiments presented here, we could dramatically affect what looks like the "academic ability" of students, simply by using alternative representations of the same information. We could increase or decrease students' ability to perceive chemistry molecules, to understand the function of hardware units, to solve text editing problems, and both to find and to solve data problems. Note that the alternative representations effect has considerable generality, for it extends across several cognitive processes (perception, understanding, problem solving, problem finding) and across several content domains (chemistry, physics/engineering, text editing, and data analysis).

Implications for Understanding and Fostering Academic Talent

The alternative representations approach holds some interesting implications for understanding "talent" and for considering educational practice for both talented and (presumably) untalented students. The experiments reported above show that we can make good students *look* more or less talented in a given domain, simply by giving them certain representations of key information. Perhaps what sets talented students apart from the rest is that they already know how to represent concepts in many ways and can select appropriate representations across a variety of tasks. Thus, appropriate representations may provide a general key to deeper comprehension and insight.

This somewhat unorthodox view of talent can be understood more fully using an analogy from ethological research. Ethologists observe animal behavior in naturalistic situations to determine what triggers responses. For example, male three-spined stickleback fish threaten other males who enter their territory by going into a characteristic attack position (assuming a vertical position with head down, tail up). It turns out that only a small part of the intruder elicits the attack position--its red underbelly (Tinbergen, 1951). In the language of the ethologists, the red "releases" attack behavior. Humans are obviously much more complex than fish and certainly respond to a richer array of information in their

environment. Yet perhaps the ability to use alternative representations similarly "releases" or "unlocks" the complex processes of comprehension and insight. If so, then instruction might well include alternative representations of important concepts and thereby identify more individuals as talented.

Using alternative representations during instruction may well facilitate comprehension and insight across academic domains. If students thereby achieve some initial success with course material, perhaps they will not fear science, may become truly engaged with the ideas, and perhaps even consider pursuing a scientific career. It may also help to provide instruction about alternative representations themselves (such as sentences, lists, outlines, trees, flowcharts, graphs, and pictures) to identify their general properties and describe how they can affect various tasks. Then students can more judiciously select representations to facilitate finding problems as well as solving them, and, in general, facilitate their ability to think academically and scientifically. In this way, we may discover that more people have the potential for academic and scientific "talent" than we currently recognize.

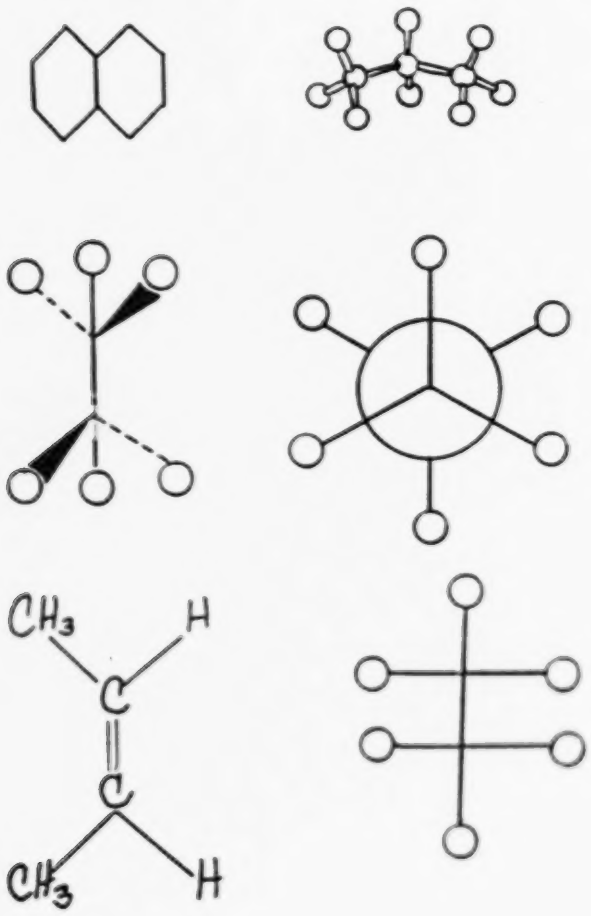
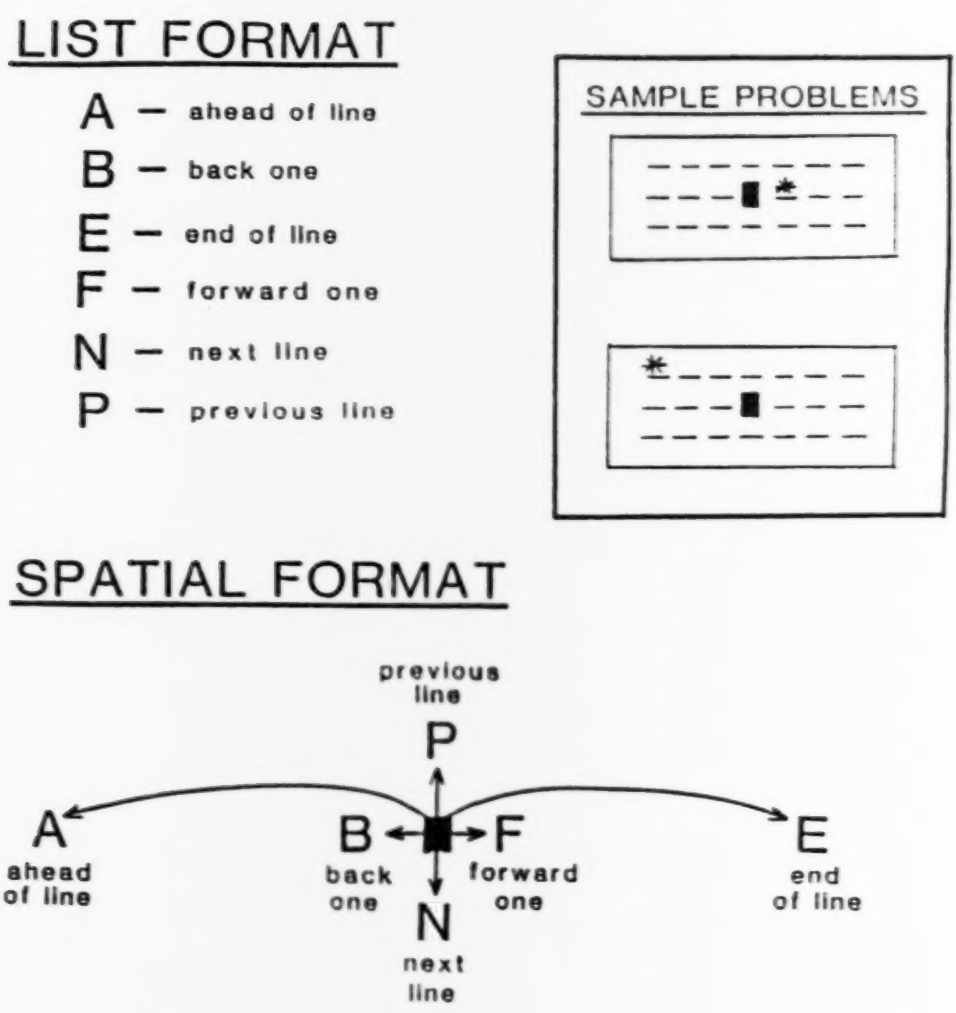


Figure 1 Representations used to display three-dimensional chemistry molecules in two-dimensional space. These representations (and others) are commonly used in organic chemistry textbooks.

Figure 2


LIST FORMAT
A – ahead of line
B – back one
E – end of line
F – forward one
N – next line
P – previous line
SAMPLE PROBLEMS
SPATIAL FORMAT
previous line
P
A
ahead of line
B
back one
F
forward one
E
end of line
N
next line

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Discussant Reaction: Cognitive Theory of Academic Talent

Edward F. Zigler

We have just heard a verbal presentation of what I found to be an absolutely excellent paper. But I'm supposed to be critical and analytic. Since I have wrestled with these same ideas for a good long while, I think I will present some old and some new thoughts that came to my mind as I read the paper. In certain ways you are going to find my views somewhat more optimistic than those of Perkins, and in other ways more pessimistic.

In a way, the paper is disappointing because the person seemed impassible. There is no mention of one of the most interesting developments in the field of intelligence, namely the social context in which cognitive development takes place. We are learning that people in different societies, different cultures, and different subcultures within a society exhibit very different intellectual profiles. Absolutely fascinating work has been done by Cole, Scribner, and Bruner. In fact, I advise Dr. Perkins that when he returns to Harvard he might chat with Gerry Lesser, who did that wonderful work some years ago which demonstrates profiles of abilities within different ethnic groups. This approach takes us away from the notion of superiority or inferiority, and takes us to where we probably ought to be in the biologically oriented behavioral sciences, namely as profile theorists. Actually, Regina Yando, Vicki Seitz, and myself advocated such an approach several years ago in a monograph.

Not only did I not find the social context, I didn't find the biological context. I do think that the future study of intelligence is going to focus more and more on the biology of learning and the biology of intelligence. It's been a long time since we read in Hebb's textbook about the notion of intelligence A, and we're still trying to unravel the phenomenon of capacity. I believe in capacity, in part because of my many years of working in the area of individual differences. Although there's been a longstanding argument over whether or not capacity is worth having as a construct, I've come to agree with Brendan Maher, who says that it's an easily operationalized construct. The phenomena is as follows: you give two people with different capacities the exact same environmental input, and one performs differently than the other. That's the operational definition of what we refer to as "capacity." Some of the most interesting work being done on this today is in the area of behavior genetics. When we look at the work of people like Sandra Scarr and Bob Plomin, and we see their findings on variations within families, I sense that we're beginning to close in on the aspects of what intelligence and its development are all about.

We have today three paradigms of intelligence to work with: 1) the psychometric paradigm; 2) the cognitive/developmental paradigm; and the most recent newcomer to the group, 3) the information-processing paradigm. If one simply takes the paradigms we have now, one can probably find approaches capable of directing current pragmatic efforts such as we are faced with here at the Talent Identification Program (e.g., research on selection procedures). But it still isn't that easy. Each of these approaches contains too many unresolved issues, too many points of contention for us to satisfy everyone.

For example, alluded to in Perkins' paper is the work of Jensen on *g*. Jensen is now defining *g* as fundamental efficiency of the neurological system. This is

surprising, as though we've come full circle back to Galton, who worked with reaction times and was trying to develop measures that were independent of learning and tapped into the physiological nature of the organism. Later there came the old Ertl work with the flicker fusion test, which was trying to accomplish the same thing. Actually, there is some interesting work in this realm going on today. My hunch is that there will probably come a time in the not too distant future when we will no longer use intelligence tests. We will find some way of assessing the quality of the central nervous system. If this seems farfetched, I suggest that you look at the recent work of Hans Eysenck who says that he already has such a measure now. I'm not as convinced as he is, but at least he's on the right track. Willerman of Texas is doing much the same thing. We are some day going to be able to get away from the bias in tests simply by tapping into a person's physiology, to get at what Hebb called passive intelligence A.

I must confess that I was taken aback by the amount of space that Professor Perkins gave in his paper to the *g* versus *s* argument. I think that it is way too late in the game for people to be arguing *g* versus *s*. I find myself in the camp with George Miller and Sandra Scarr, each of whom have written devastating critiques of Howard Gardner's work. We are now aware that there is *g* and that there are also *s* factors. We have positions in the middle, like Vernon's with his hierarchical group factors. All of this is now known, and to bring it up again when we have so much else that we can legitimately study and debate is a waste of time and energy. They are both right, and any complete model will have to incorporate both of them.

The problem with *g* and *s* is that they are tied to the psychometric paradigm. They come out of tests and the analysis of tests. Unfortunately, we've convinced ourselves that the psychometric approach and the cognitive/developmental approach are different. The fact is, they are not that far apart. Using different types of measures, they both tap into the cognitive processing of the individual. While the argument goes on, the work by Kaufmann and others indicates that the correlation between those two types of measures is about .7, which is pretty high.

As for my colleague Bob Sternberg's triarchic model, and I certainly have a great deal of respect for Bob, I feel that it is simply too early in the day to understand just what meaningful information it is going to add. I do agree with David Perkins' assessment that the triarchic model may not be any more predictive than the psychometric measures we already have. However, this is an empirical issue, and Bob Sternberg has a new test forthcoming, so we'll have to see what the results are. I refer you to a very telling critique of Bob Sternberg's theoretical formulations by Neisser at Emory, who calls into question whether or not the triarchic formulation has the formal characteristics of a theory.

And now I slip into my own predilections and my own prejudices. I number myself among those who've taken the cognitive developmental approach to intelligence, its genesis, and its growth. I think that David Perkins is probably right. Of late Piagetians have fallen on hard times. But they're not nearly as hard as Professor Perkins would have us believe, so it's premature to sound the death knell on Piaget. The fact is that ever since John Flavell's presidential address at SRCD, we have been aware that the system works better in individual

domains than it works across domains. Problems of horizontal and vertical decalages have caused problems with the overall theory, as David pointed out. However, we're working on these, and I continue to find a great deal of value in thinking about the human being as moving from stage to stage to stage, not only in cognitive development but in physical and emotional development as well.

One difficulty is that Piaget has captured the field so completely that many thinkers are not aware that there are other developmentalists who have made equally valuable contributions to our understanding of intelligence. I think in particular of Vygotsky, who emphasizes the forgotten social context. Also my own mentor, Heinz Werner, and his three-stage model of global to differentiated to hierarchic integration. His model is a precursor of modern work. For example, not very many miles from Harvard is my old colleague Bernie Kaplan, whose book represents how Werner's model fits language development. I myself have been using developmental models for a long time to try to comprehend individual differences. They make it possible for me to quickly shift my attention from the retarded to the gifted group, because the relationship of retarded to average functioning is exactly the same as the relationship of the average to the gifted level.

I think that there are two very important phenomena which illuminate what we've been talking about this afternoon. They give us some notion of where to look for precursors of giftedness as well. One has to do with the rate of cognitive development. For an operational definition of stage of development, I've used the MA (Mental Age) for 30 years. We know that people move through MA stages at very different rates. Someone moving through these stages very rapidly is a potentially gifted individual. There's nothing brilliant about someone observing that if you cut a pie into two uneven parts, the sections are not halves. What's special is that it is noticed by a six year old. If a 15 year old said that, we wouldn't be impressed. This brings me back to our old friend Galton--everything seems to double back to Galton. He was a phenomenal little child. He could read and write at the age of four, and attained a scholarly understanding of *The Iliad* and *The Odyssey* by age six. The people who knew young Galton didn't think there was anything amazing about him, since any respectable school graduate could do what he did. What was amazing, however, was that he was so young to achieve so much. Traversing the stages of development very rapidly is item one to look for in a precursor.

The other phenomenon can't be used as a precursor; it has to do with individual differences. People do not wind up at the same final level of cognitive development. Plenty of people never make it into logical operations. And now we're talking about a post-logical operations stage. So what we're looking for is people who go through the stages very rapidly and wind up at an asymptote that's much higher than average. These are our gifted individuals. I have just begun testing a model on Duke TIP students. If my model is right, we should be able to compare gifted people with older individuals who are their MA mates, and show that their cognitive functioning and motivational characteristics are just about the same. We've done one study that supports this hypothesis, so at least we're on the right track.

Let's move to another problem, the creativity versus intelligence issue. This has also been around for a good long while, so we're all fairly conversant with it. I find myself pretty much in agreement with Perkins that it looks like intelligence is necessary for creativity, but not sufficient. Still, if something is necessary, it at least gives us a place to start. That's why when we look at giftedness we continue to have to rely on what's necessary, namely, a very high IQ. Unfortunately, we've never been able to get a very good handle on creativity. There is a body of work which cuts across the psychoanalytic camp, the cognitive camp, and the developmental camp. I refer you to a book by Arieti, *The Magic Synthesis*, where he attempts to show that a combination of some earlier forms of thinking, in line with the ego in his psychoanalytic term, gives you creativity. If you picked up Kessler's book, *The Act of Creation*, you could find that same appeal, that creativity is thinking in very divergent, unusual kinds of ways.

Developmental thought gives us some leads here. There are certain phenomena of thought that characterize the very young child which are lost in later development. It would be wonderful if we could access them. Synesthesia is a good example. In research on people's perceptions of the color of the sound of a trumpet, we find that every kid is better at synesthesia than every adult. This and other phenomena seem to be characteristics of the early forms of development which somehow drop out of the system. Werner and other developmentalists have said that the combination of the lower forms with the higher forms results in a creative individual. If you just use the lower forms, the result is a schizophrenic. In Piaget's telescope model of development, those early forms don't just vanish, they are embedded within later forms. If we could find some way to break those early forms out and make them accessible so they could operate in combination, we'd have a storehouse of creativity.

I've always been troubled by the assertion that IQ tests don't measure creativity, because most definitions of creativity sound very much like what we ought to be saying intelligence is. I find myself in agreement with Quinn Nemar. In his presidential address to the American Psychological Association many years ago, he made a point that I've made in some of my writings as well: we need measures of creativity in our intelligence tests. The problem is that creativity is a very unusual, very rare event. A highly creative person is very rare, and the standard measures that we construct are not very good at signifying these rare events. They're good at gettng everyday events. So this means that there's still plenty of room for people to try and study what we mean by intelligence and what measures we need to assess it. Our first sound intelligence tests, beginning with Binet, are really very arbitrary. We've been almost victimized by their success. We can look at their failures, as David Perkins has chosen to do, and think about new measures that will encompass creativity.

One other phenomenon that I found inadequately covered in Perkins' paper was the whole issue of motivation. When he does discuss motivation, he discusses it in a very thin way, by pointing to the work of Bloom. Bloom focuses on all the work and energy and effort that parents and kids put into their work. To me, that's a matter of extrinsic motivation; that's when you're programmed to become a genius. That isn't what most geniuses and creative people really look like. I would be much more interested in a discussion of what is called effectance motivation. With this impetus, people work diligently for the sheer joy of using

their own intellect. I've studied this concept, usually within the construct of humor. I've learned that we most enjoy jokes that we have to work to understand. To me, the nature of intrinsic motivation, where we have to do most of the work, rather than the kind of extrinsic motivation that Bloom and Perkins have emphasized, brings us closer to what giftedness is all about.

We've heard from both David Perkins and my old colleague Ruth Day about efforts to make people perform more efficiently. This is certainly possible. There are skills and pedagogy which improve performance. Of course, those of us who've been around for a while remember Moore's talking typewriter and the ten thousand other things that have come along in the last 30 years which guarantee to make everybody smart. I've become pretty skeptical. I think about the work in mental retardation of my colleagues Butterfield and Belmont in Kansas. They were working on that little monkey list that shows up in Perkins' paper, but there was very little generality. We were not successful in making retarded individuals much smarter. There is a new book by Herman Spitz, who reviews 100 years of efforts to make retarded persons more intelligent, all of which have failed. Does that mean we quit trying, that we don't try new inventions? No, of course not. To me, these are empirical issues. I'm just saying that I need hard evidence before I'll be terribly convinced that people can be trained to be much smarter in any general sense.

This brings me back to the precursor issue. David Perkins seems to be saying that we have in our TIP students saplings, but he would like to get back to the seed. I look to the work of behavior geneticists and say that we don't even have to wait for the seed. We know the best predictor of an individual's intelligence before he or she ever arrives at Duke, or comes into this world for that matter. That is the midpoint of the parents' IQs. It isn't a perfect measure, of course, and there is still a lot of room for environmental influence. But, it is simply too strong an indicator to ignore.

I want to pick up the competitiveness issue. Should we start approaching intelligence and giftedness and creativity the way we approach basketball players? We're all aware of values that were discussed in Al Trivelpiece's keynote address. We're all very sensitive to the charge of being elitists. There are two responses to this charge. One I reject out of hand, and that's the one of people like Renzulli and others who say that everybody is intellectually gifted and let it go at that. I don't think that's acceptable. I'm so immersed in a lifetime of studying and respecting individual differences, wherever they might be, that I simply cannot tolerate such a homogenization of the human race. My hunch is that we're going to be stuck with the charge for awhile until we get better measures of IQ, so the second response is just to wait. We can try measures that TIP uses, like the SAT, which is probably correlated respectably with IQ. That doesn't mean we shouldn't continue to look for better measures. If we don't, then I think the charge of elitism is appropriate.

We can refute this charge right now if we commit ourselves not just to the gifted, but to that vast array between giftedness and retardation where most of humankind is. My knowledge as a developmentalist who has learned something about cognitive psychology tells me there's absolutely no reason on this earth why children shouldn't be learning to read by the age of 7, why the dropout rate is so

high, or why schools are doing such a poor job with our young people. It is imperative that we think in terms of upgrading education for all children in the U.S. By so doing, we will allow a higher number of children to exercise their giftedness. We will also guarantee a generation who can read and who are literate in math and science. This is every bit as important as having that handful of great intellectual leaders.

I would also like to make a comment that I've stolen some from my colleague Fran Horowitz. We must not think about children solely as little cognitive systems and worry about how to maximize those systems so we can have great mathematicians and scientists in the next generation. We must recognize that education and cognitive development sit within the body of a child who is much more than this. If you really want to produce more geniuses and better kids in general, you're going to have to look beyond education and whiz-bang intervention efforts to make kids brighter. You're going to have to look at what kind of child care they are receiving. You're going to have to look at what kind of health care they get, what kind of nutrition they have, and how strong their families are. So beyond focusing on the whole population of children, we must focus on the whole child.

I would like to bring up another concern I have about what happens to the curiosity, the inquisitiveness that is natural to the very young child. All of us who work with children at the ages of 3, 4, and 5 know that everything is a question to them. They're always looking for problems, they're always looking for things to take apart. Give them something they can't take apart, and they'll find a way to take it apart a 12 year old wouldn't begin to try. What happens? I'm a veteran of the post-Sputnik era, when Admiral Rickover said, "Of course the Russians are ahead of us. They teach their kids engineering and their school days are longer. All we're doing with our children is teaching them finger painting, eight months a year. We've got to get back to basics. More reading, writing, arithmetic. Stop this gym, this acting, stop all the things that enrich the mind and the human being."

Again, I'm indebted to Fran Horowitz who suggested that we've come this way before. As I look about the country, I find that those who don't know history are forced to relive it. We are reliving it. The response of this nation to the first report, *A Nation at Risk*, and the dozen that came after it has been exactly what happened after Sputnik. How do you make the days longer? How do we keep kids in school more days a year? How do we get back to the "hard core" effort? We're even seeing this emphasis come down for very young children. I attended a conference at Yale a few months ago which resulted in a book by Lynn Kagan and myself called *Early Schooling: The National Debate*. Today we are seeing some schooling for four year olds that looks like the first grade has been moved down to age four. The play, the curiosity, the self-discovery of children are being wiped out by people who think that the only way you can teach youngsters is to sit them at desks and yammer at them, or have them fill out little workbooks. If you want to know exactly what I'm talking about, I refer you to the book that says it all in the title--David Elkind's *Miseducation*.

Let me conclude by saying one final thing about which I'm somewhat pessimistic. One of the big problems we face is the problem of school change.

We have some very brilliant workers working on this issue, but it's still far from solved. I'm friendly with some of them. I just came back from a meeting in Seattle with John Goodlad, who's one of the great thinkers about how to get schools to change. Seymour Sarason, my colleague at Yale, has been writing about this for a good number of years now, and pointed out why the new math fell on its face. Part of the problem is that there are sixteen thousand school districts in the country. How in the world do you bring change into a polyglot system of that kind?

My position is that if we had leadership--not a school basher, but a real leader--helping schools do what schools ought to be doing, I think we would see progress. The system does work. When I was in Washington as Chief of the Children's Bureau, I decided it would be a good idea to have education for parenting in schools. I was very worried then about teen-age pregnancy. I thought that teaching kids what parenting was all about might help. We developed a curriculum, distributed it, and it's now used in 3000 schools in America. It's not in all 16,000 districts, but 3000 isn't bad. In New Zealand, exactly the same curriculum is used in every school. Apparently, it's much easier to disseminate nationwide programs when you have a Ministry of Education. I believe we could have the same success in the U.S. with the proper leadership.

I want to end on an up note. I have been reading Arthur Schlesinger's new book in which he points out that societies have waves of progress and regressions, zeniths and nadirs and back. Those of us who have seen the good, innovative times of the '60s, and the very bad times of the '80s might take some heart from Schlesinger. This great historian's view suggests that America may be on the verge of another upward swing. We heard from our friends in Washington that the window is open; I believe that's right. We're all going to have to work very hard to try and scurry through that window while it is open, but I am optimistic we will be there on time. In conclusion, I think it's appropriate for me, on behalf of the audience, to thank David Perkins for an absolutely excellent analysis of the problem that we are here to face.